Maximizing Welfare for Development Recipients with Multi-criteria Analysis

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Choices are the hinges of destiny

– Edwin Markham

An Introduction from Travis Watters:

For most of my childhood, a stately array of cereal boxes spanned the entire first row of the kitchen pantry. I was never an early riser and a simple bowl of grain, sugar, milk, and sprayed-on vitamins was perhaps the best compromise between my desire for sleep and my parents' desire to nourish me. Each day, I would choose from quite a few alternatives, weighing several criteria. For taste, Frosted Flakes was king. For nutrition, Total Raisin Bran shared no equal. For back-of-box reading material, Cinnamon Toast Crunch held the greatest appeal. I never developed an algorithm for making the best selection, but within 15 seconds, I always ended up chomping down and solving mazes through heavy-lidded eyes.

At Tetra Tech, in my role in international development consulting, I'm constantly faced with evaluating a suite of alternatives across multiple criteria, where the stakes are as high as billion-dollar foreign aid investment packages for countries facing evolving water scarcity challenges. How do we decide? This question lies at the heart of a field known as Multiple Criteria Decision Analysis, or MCDA.

In consultation with my co-author Christopher Tofallis, Associate Professor of Decision Science at the University of Hertfordshire in England, we broach how MCDA can be used to select a foreign aid package using a hypothetical scenario for the mythical country of Yellow Cactus Island.

THE LABYRINTH OF MULTIPLE CRITERIA DECISION ANALYSIS

Yellow Cactus Island, a fictitious island nation, is experiencing constraints to economic growth due to impending and progressive water scarcity and geographic isolation. In partnership with a potential donor, a team of engineers, scientists, economists, and decision makers need to employ MCDA to discern which investment will return the highest dividends. Synthesizing several definitions, we will use the framework shown in Exhibit 1 below to decide which evidence-based intervention to finance.

Exhibit 1:

1. **Identify the goal** and generate alternative solutions for meeting it
2. **Identify criteria** for comparing the efficacy of the alternatives in meeting the goal
3. **Attach numerical measures** for expressing the "goodness" or "badness" of the alternatives in each of the criteria; if required, normalize these numerical measures to place them on comparable scales
4. **Assign weights** to the relative importance of the criteria in meeting the goal (weighting)
5. **Aggregate** these numerical values to rank, group, or dismiss alternatives

As we endeavor to use MCDA to develop the best suite of projects for maximizing the Yellow Cactus Islanders' welfare, we will encounter a maze far more treacherous than any found on a cereal box. Our journey will be more akin to the venture of Theseus, descending into the labyrinth with a ball of yarn, aiming to slay the dreaded Minotaur; and we shall attempt to navigate our course with the same cunning and skill.

Before we begin, we must acknowledge a dilemma, known as the decision-making paradox, where different decision-making methods yield different results, even when fed the exact same problem and data. The choice of method is critical, yet the mechanism for determining the best method is unclear.

For the sake of the Yellow Cactus Island's sanity, we will restrict ourselves to pursuing one of the simplest methods, known as the Weighted Product Method (WPM). In WPM, each alternative receives a score for each criterion. Each score is raised to the power of its weight and the scores are multiplied to obtain a total Performance Score. "More-is-better" categories, like number of new crops introduced, are given a positive exponent. "More-is-worse" categories, like cost, are given a negative exponent. In WPM, re-scaling the values, e.g. by changing measurement units, has no effect on the outcome. And, while many other methods require normalization—a procedure wherein all values are transformed to make them comparable in some sense—WPM does not require such standardization; this is a valuable feature as different normalization approaches lead to different outcomes.

Now, let us content ourselves with WPM for the purposes of our current exploration. For the mathematically inclined reader, the method can be stated as follows in Exhibit 2:

Exhibit 2:

$$A_{WPM} = \prod_{j=1}^{n} (a_{ij})^{w_j} \qquad \text{For } i = 1, 2, 3, \ldots, \boldsymbol{M}$$

Where:

$\boldsymbol{M}$ is the total number of alternatives

$\boldsymbol{n}$ = the total number of criteria or categories

$\boldsymbol{i}$ = the particular alternative being evaluated

$\boldsymbol{j}$ = the particular criterion being evaluated

$\boldsymbol{A_{WPM}}$ = the overall performance score of alternative $\boldsymbol{i}$;

$\boldsymbol{a_{ij}}$ = the score of alternative $\boldsymbol{i}$ on criterion $\boldsymbol{j}$;

$\boldsymbol{w_j}$ = the weight for criterion $\boldsymbol{j}$

If this mathematical Minotaur is spiking your anxiety, try a deep, relaxing breath. We'll step through this slowly when the time comes. For now, let's celebrate our triumph and enter into the maze.

STEP 1: IDENTIFY THE GOAL

The goal is often defined in a deliberately vague fashion, to contextualize how the program or investment fits into the big picture. Within this lofty goal, we can identify a wide swath of alternatives and criteria that become the steppingstones to the vision. Let us state, then, that our goal is to select the investment package that maximizes the welfare of the citizens of Yellow Cactus Island within the constraints of time and budget.

For the second portion of this step — generating solutions — let us content ourselves with terming the alternatives A, B, C, and D. These alternatives could be anything: development of educational programs; construction of water systems, roadways, or electrical facilities; modernization of governmental communication networks — you name it.

This step is unique to each situation and must involve consultation with host countries, funding agencies, and consultants.

STEP 2: IDENTIFY CRITERIA

To identify the right criteria for Yellow Cactus Island, we must abide by a set of assumptions. In general, criteria must follow (at least) these rules:

1. **Criteria should be independent of each other to distinctly measure the impact of the outcome.** Some relationship between criteria is expected by nature but the values of one criterion should not be scale multiples of another. Considering two such criteria is equivalent to double-counting the underlying criterion, and will skew our results.
2. **Criteria should materially impact the stated goal.** When conceptualizing your design, you must generally hypothesize that your criteria will advance your goal. If that expectation or prediction does not exist, that criterion doesn't deserve a stake in the evaluation.
3. **Criteria should account for the full benefit or cost to the overall goal.** Considering the costs and benefits over the entire life-cycle of the investment can be a good way to make sure criteria are given their proper due.
4. **Taken together, the criteria considered should account for all aspects of the program and its intended impact as simply as possible.** A proliferation of criteria complicates the analysis, but we can't use this excuse to leave out critical aspects. The criteria we select for evaluation should be the smallest set that encapsulates the issues we most care about.

Now, to generate criteria for our analysis we must first think critically about our options. Let's say that we are proposing water infrastructure improvements that will improve sanitation and curb widespread transmission of waterborne illness. For the purposes of this example, we will consider our criteria to be cost, economic improvement, social impact and gender equality, environmental benefit, public health benefit, and climate change. We will measure these using the variables defined in table 1.

With these criteria in hand, we may put forth a skeletal investment decision matrix, which we might consider our "Map" through the labyrinth. There's plenty of data yet to gather, but the blueprint is starting to take shape *(shown in exhibit 3)*:

Exhibit 3: *Skeletal Decision Matrix for Yellow Cactus Island Investment Packages*

ALTERNATIVES	1 Life-Cycle Cost, in 2020 USD	2 Job-Days Created	3 Estimated Gender Wage Gap Closure (Δ Participants) (Δ Average Salary) (2020 USD)	4 Average Contaminant Concentration Decrease in Surface Waters (mg/L)	5 Diarrheal Disease Incidences Averted	6 Lifetime Tons of Greenhouse Gases Added to Atmosphere
A						
B						
C						
D						

STEP 3: ASSIGN NUMERICAL MEASURES

In the real world, Tetra Tech's first step in assessments of this kind often involves data acquisition through surveys, sampling efforts, interrogation of existing databases, and the like. This effort is often coupled with a predictive analysis in terms of cost estimates and the likely impact of proposed interventions on the country's existing condition, relative to the criteria of interest of course. Let us assume that our intrepid forces have completed these steps for Yellow Cactus Island and have used these investigations to begin filling in our decision matrix *(shown in Exhibit 4)*.

Exhibit 4: *Criteria Values for Yellow Cactus Island Investment Packages*

ALTERNATIVES	1 Life-Cycle Cost, in 2020 USD	2 Job-Days Created	3 Estimated Gender Wage Gap Closure (Δ Participants) (Δ Average Salary) (2020 USD)	4 Average Contaminant Concentration Decrease in Surface Waters (mg/L)	5 Diarrheal Disease Incidences Averted	6 Lifetime Tons of Greenhouse Gases Added to Atmosphere
A	13M	50,000	600,000	2.5	100,000	140,000
B	17.5M	40,000	480,000	2.5	80,000	110,000
C	12.5M	44,000	528,000	3	250,000	150,000
D	15M	60,000	720,000	2.8	50,000	180,000

STEP 4: ASSIGN WEIGHTS

Suppose that we decided that life-cycle cost is more important to our decision-making coalition than job-days created by a factor of approximately 3. We would apply a weight factor to both categories to capture this trade-off. How best to perform that assignment is the subject of debate in the field and no small part of the reason for the impressive proliferation of MCDA methods. For illustrative purposes, we will assign weights by choosing a base reference criterion — in this case cost. Adjusting the weight for any one criterion implicitly defines relationships between all other criteria, so the procedure does not need to be repeated.

Thus, somewhat arbitrarily, we arrive at the following weights:

- A **3 percent increase** in job-days is worth a 1 percent increase in cost (exponent = 1/3)
- A **10 percent increase** in wage-gap closure is worth a 1 percent increase in cost (exponent = 1/10)
- A **3 percent decrease** in average contaminant concentration is worth a 1 percent increase in cost (exponent = 1/3)
- A **0.4 percent increase** in diarrheal diseases averted is worth a 1 percent increase in cost (exponent = 1/0.4 = 5/2)
- A **2 percent reduction** in greenhouse gases is worth a 1 percent increase in cost (exponent = -½)

And we complete Exhibit 5:

Exhibit 5: *Complete Decision Matrix for Yellow Cactus Island Investment Packages*

ALTERNATIVES	1 (W_1=1) Life-Cycle Cost, in 2020 USD	2 (W_2=1/3) Job-Days Created	3 (W_3=1/10) Estimated Gender Wage Gap Closure (Δ Participants) (Δ Average Salary) (2020 USD)	4 (W_4=1/3) Average Contaminant Concentration Decrease in Surface Waters (mg/L)	5 (W_5=5/2) Diarrheal Disease Incidences Averted	6 (W_6=1/2) Lifetime Tons of Greenhouse Gases Added to Atmosphere
A	13M	50,000	600,000	2.5	100,000	140,000
B	17.5M	40,000	480,000	2.5	80,000	110,000
C	12.5M	44,000	528,000	3	250,000	150,000
D	15M	60,000	720,000	2.8	50,000	180,000

STEP 5: AGGREGATE

All right, it is finally time to slay the mathematical Minotaur. Let's take that gnarly Equation (Exhibit 2) and step through it just a bit at a time.

We'd like to get an aggregated performance score, AWPM, for Alternative A. We know our weights (exponents) for each criterion. We know cost is bad, greenhouse gas emissions are bad, and everything else is good, so we fill in our numerator and the denominator accordingly as shown in Exhibit 6.

Exhibit 6:

$$A_{WPM} = \prod_{j=1}^{n} (a_{ij})^{w_j} = \frac{(50{,}000)^{1/2}(600{,}000)^{1/10}(2.5)^{1/3}(100{,}000)^{5/2}}{(13{,}000{,}000)^{1}(140{,}000)^{1/2}} = 123{,}000$$

We repeat this step for all remaining alternatives and derive the values shown in Exhibit 7:

Exhibit 7: *Performance Scores and Ranks for Yellow Cactus Island Investment Packages*

	A	B	C	D
Score	**123,000**	**54,000**	**1,227,000**	**19,000**
Rank	**2**	**3**	**1**	**4**

Hard to believe all our effort reduced to that! But that's exactly what we wanted; something that gives us a simple, understandable way of aggregating a messy clot of data. And as a matter of fact, that's exactly what Tetra Tech does — take a complex world and create clear solutions.

It's often useful to perform a "robustness" check, or a sensitivity analysis, on this outcome. That is, how much would we have to perturb our scores or weightings before we got changes in ranking? Tryantaphyllou has put forth a method for performing this check, though we will leave that for further exploration. This robustness check can also help us refine our approach.

MCDA is usually iterative; sometimes, we get results that seem drastically counter to our expectations. This is usually a good sign that we've failed to account for or appropriately weight something that's important to us. We may find ourselves somewhere in the maze we didn't expect, and we have to decide whether this was our destination all along, or whether we should retrace our steps and re-try, perhaps this time with a different approach.

CONCLUSION: OUT OF THE MAZE!

For now, here ends our journey through the labyrinth, with what is hopefully an important lesson: there is no one way to solve the maze. In fact, at present, there is no way to be certain you've reached the center! No single method of MCDA dominates all others in the field, and we must exercise great care at each of the crossroads encountered as we venture through the forking paths. It is more correct to say that we gain confidence in our answer as we iterate through multiple routes and find ourselves at the same destination time and again. It is no simple feat! But they don't write legends about people who only solve the mazes on the backs of cereal boxes, do they?

For the citizens of Yellow Cactus Island, we have selected from among a suite of investments a package that, ostensibly, meets our goal of maximizing the welfare within our budgetary constraints. This evidence-based approach gives us confidence in our choices, and also casts an eye to the future in terms of what metrics we should be monitoring to evaluate our success. This feeds into our knowledge base and sharpens our senses for the next analysis, in an ever-improving decision-making process.

BIO SUMMARIES

Travis Watters is an project manager at Tetra Tech with 13 years of experience in hydraulic and hydrologic modeling, planning, and design of water, stormwater, and wastewater distribution systems. While with Tetra Tech, he has supported international development projects funded by USAID and MCC in Afghanistan, Ethiopia, Haiti, Liberia, Mongolia, the Philippines, Timor-Leste, and Zambia.

Dr. Chris Tofallis is a researcher and lecturer in Decision Science at the University of Hertfordshire, England, with expertise in multi-criteria decision analysis, performance and efficiency measurement based on multiple criteria, quantitative forecasting, and data fitting. His research is aimed at developing data-analytic methods which are based on simple principles but which provide advantages over existing approaches.